PUNJAB PUBLIC SERVICE COMMISSION

COMBINED COMPETITIVE EXAMINATION
FOR RECRUITMENT TO THE POSTS OF
PROVINCIAL MANAGEMENT SERVICE, ETC -2022
CASE NO. 2C2023

SUBJECT: MATHEMATICS (PAPER-I)

TIME ALLOWED: THREE HOURS **MAXIMUM MARKS: 100**

NOTE:
- i. All the parts (if any) of each Question must be attempted at one place instead of at different places.
- ii. Write Q. No. in the Answer Book in accordance with Q. No. in the Q. Paper.
- iii. No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
- iv. Extra attempt of any question or any part of the question will not be considered.

NOTE: **Attempt FIVE Questions in All including THREE questions from Part-A and TWO questions from Part-B. Calculator is allowed. (Non-Programmable)**

PART-A

Q.No.1 (a) For what values of a, m, and b does the function

$$f(x)=\begin{cases} 3, & x=0 \\ -x^2+3x+a, & 0<x<1 \\ mx+b, & 1\le x\le 2 \end{cases}$$

satisfy the hypotheses of the Mean Value Theorem on the interval [0, 2]?

(b) Find the asymptotes of the curve $2xy+2y=(x-2)^2$.

(10+10=20 Marks)

Q.No.2 a) Evaluate $\int \frac{\sin x}{\sin 3x}\,dx$

b) The velocity of a car travelling on the Motorway at 15 minutes intervals is as follows:

Time in hours, $t=$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{5}{4}$
Velocity in km/hr, $v(t)=$	100	90	115	120	80

Find the total distance travelled by the car in the 60-minute period from $t=\frac{1}{4}$ to $t=\frac{5}{4}$.

Q.No.3 (a) Find the area of the region bounded by the curve $y=4x-x^2$, the x-axis, and the lines $x=1$ and $x=3$.

(b) Using rectangular rule for $n=4$, approximate the value of the definite integral $\int_0^1 \frac{dx}{1+x^2}$.

(10+10=20 Marks)

Q.No.4 a) Discuss the motion of a particle moving in a straight line if it starts from rest at a distance a from a point O and moves with an acceleration equal to μ times its distance from O.

b) A culture initially has P_0 number of bacteria. At $t = 1$ hour the number of bacteria is measured to be $\frac{3}{2}P_0$. If the rate of growth is proportional to the number of bacteria $P(t)$ present at time t, determine the time necessary for the number of bacteria to triple.

(10+10=20 Marks)

Q.No.5 **(a)** Solve the differential equation $\frac{dy}{dx} = (-2x + y)^2 - 7;\quad y(0) = 0$.

(b) A particle of mass m oscillates in a line with natural period $\frac{2\pi}{\omega}$. If an applied force $F\cos pt$ now acts in the line so that the particle is instantaneously at rest at zero time at a distance d from the centre of oscillation, prove that the displacement of the particle from the centre at subsequent time t is $d\cos\omega t + \frac{F(\cos pt - \cos\omega t)}{(\omega^2 - p^2)m}$.

(10+10=20 Marks)

PART-B

Q.No.6 **(a)** The nth term of a sequence is $n^{1/n}$. Determine whether sequence converges or diverges.

(b) Determine the convergence or divergence of the series $1 + \frac{1\cdot 2}{1\cdot 3} + \frac{1\cdot 2\cdot 3}{1\cdot 3\cdot 5} + \frac{1\cdot 2\cdot 3\cdot 4}{1\cdot 3\cdot 5\cdot 7} + \ldots$ by applying any appropriate test.

(10+10=20 Marks)

Q.No.7 a) Prove that the Necessary and sufficient condition for a function $W = f(Z) = U(x,y) + i\,V(x,y)$ to be an analytic function is that the four partial derivatives U_x, U_y, V_x, V_y exist, are continuous and satisfy the Cauchy Riemann equations at each point of D_f i.e., $\frac{\partial U}{\partial x} = \frac{\partial V}{\partial y}$ and $\frac{\partial U}{\partial y} = -\frac{\partial V}{\partial x}$.

b) Prove that the function $f(Z) = \sqrt{|xy|}$ is not analytic at the origin although the Cauchy Riemann equations are satisfied at the origin.

(10+10=20 Marks)

Q.No.8 **(a)** Find the equation of the osculating plane to the Helix

$$x = a\cos\theta, \quad y = b\sin\theta, \quad z = b\theta \qquad \text{at } \theta = \frac{\pi}{2}$$

(b) Show that the radius of curvature ρ and radius of torsion σ of the curve $\vec{r} = (a\cos u,\ a\sin u,\ a\cos 2u)$ at $u = \frac{\pi}{4}$ are $\rho = 5a$ and $\sigma = \frac{5a}{6}$.

(10+10=20 Marks)

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SUBJECT: MATHEMATICS (PAPER-II)

TIME ALLOWED: THREE HOURS

MAXIMUM MARKS: 100

NOTE:

i. All the parts (if any) of each Question must be attempted at one place instead of at different places.
ii. Write Q. No. in the Answer Book in accordance with Q. No. in the Q. Paper.
iii. No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
iv. Extra attempt of any question or any part of the question will not be considered.

NOTE: Attempt any FIVE questions. Calculator is allowed. (Non-Programmable)

Q.No.1 (a) If order of an element b is n. Then show that the elements $b^0, b^1, b^2 \dots . b^{n-1}$ are all distinct and $b^k = e$ iff k is divisible by n.

(b) Show that the only idempotent element in a group G is its identity.

(10+10=20 Marks)

Q.No.2 (a) Prove that every sub group of a cyclic group is itself cyclic.

(b) Show that any two cyclic groups of the same order are isomorphic to each other.

(10+10=20 Marks)

Q.No.3 (a) Define an integral domain. If p is a prime number, then show that ring of integers mod p is an integral domain.

(b) Let F be a field of real numbers. Then set of all real valued functions whose nth derivative exist for $= 1,2, \dots$, is a subspace of all real valued continuous function on $[0,1]$.

(10+10=20 Marks)

Q.No.4 (a) Let U and W be 2 −dimensional subspaces of $\mathbb{R}^3$. Show that $U \cap W \neq \{0\}$.

(b) Give at the least three examples of infinite dimensional vector spaces.

(10+10=20 Marks)

Q.No.5 (a) Show that the intersection of any number of topologies is also a topology and is coarser than each of the given topologies.

(b) Show that a subspace of a topological space is itself a topological space.

(10+10=20 Marks)

Q.No.6 (a) Solve

$$\frac{1}{4}x_1 + \frac{2}{4}x_2 + \frac{1}{4}x_3 = 40$$
$$\frac{2}{4}x_1 + \frac{1}{4}x_2 + \frac{1}{4}x_3 = 50$$
$$\frac{2}{4}x_1 + \frac{2}{4}x_3 = 60$$

(b) Show that the system $Ax = b$ has a unique solution if A is non-singular matrix.

(12+8=20 Marks)

Q.No.7 **(a)** Prove that

$$\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \\ 1 & 1 & 1 & 1+d \end{vmatrix} = abcd\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)$$

(b) Find the eigenvalues and corresponding eigenvectors of the matrix

$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

(10+10=20 Marks)

Q.No.8 **(a)** Find the solution of the given system of equations by reducing it to reduced echelon form

$6x_1-6x_2+6x_3=6$

$2x_1-4x_2-6x_3=12$

$10x_1-5x_2+5x_3=30$

(b) For what Value of λ do the following homogeneous equations have non trivial solutions? Find these solutions

$(3-\lambda)x_1-x_2+x_3=0$

$x_1-(1-\lambda)x_2+x_3=0$

$x_1-x_2+(1-\lambda)x_3=0$

(10+10=20 Marks)