PUNJAB PUBLIC SERVICE COMMISSION

COMBINED COMPETITIVE EXAMINATION
FOR RECRUITMENT TO THE POSTS OF
PROVINCIAL MANAGEMENT SERVICE, ETC -2021
CASE NO. 3C2022

SUBJECT: MATHEMATICS (PAPER-I)

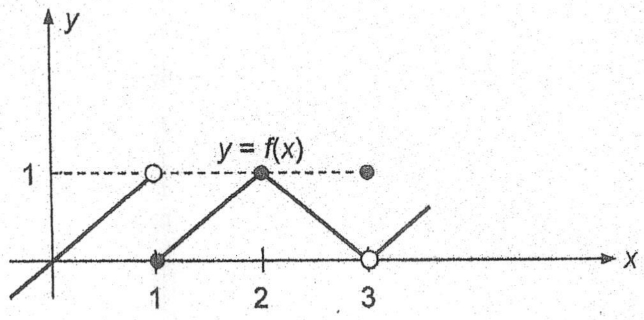
TIME ALLOWED: THREE HOURS MAXIMUM MARKS: 100

NOTE:
i. All the parts (if any) of each Question must be attempted at one place instead of at different places.
ii. Write Q. No. in the Answer Book in accordance with Q. No. in the Q. Paper.
iii. No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
iv. Extra attempt of any question or any part of the question will not be considered.

NOTE: **Attempt FIVE Questions in All. THREE Questions from Section 'A' and TWO Questions from Section 'B'. Calculator is allowed. (Not programmable)**

SECTION-A

Q.1 **(a)** For the function $f(x)$ graphed in the adjoining figure, find the following limits or explain why they do not exist.
(i) $\lim_{x\to 1} f(x)$ **(ii)** $\lim_{x\to 2} f(x)$
(iii) $\lim_{x\to 3} f(x)$
Also discuss the continuity of $f(x)$ at $x = 1$, $x = 2$ and $x = 3$.



(b) Differentiate $y = \sqrt[3]{\dfrac{x(x^2+1)}{(x-1)^2}}$ with respect to x. **(10 + 10 = 20 Marks)**

Q.2 **(a)** For what values of a, m, and b does the function

$$f(x) = \begin{cases} 3, & x = 0 \\ -x^3 + 3x + a, & 0 < x < 1 \\ mx + b, & 1 \le x \le 2 \end{cases}$$

satisfy the hypotheses of mean value theorem on the interval $[0,2]$?

(b) A box with rectangular base, whose length is twice its width, is to have a closed top. The area of the material in the box is to be 192 in^2. What should the dimensions of the box be in order to have the largest possible volume?

(10 + 10 = 20 Marks)

Q.3 **(a)** Show that $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\,dx = \frac{\pi}{4}$.

(b) Evaluate $I = \displaystyle\int_0^{4}\int_0^{4-x}\int_0^{4-x-y} dz\,dy\,dx$. **(10 + 10 = 20 Marks)**

Q.4 **(a)** A plot of land lies between a straight fence and a curved stream at distance x meter from one end of the fence, the width y meters of the plot was measured as follows

x	0	10	20	30	40	50	60	70	80
y	0	32	44	58	63	50	30	32	0

Find the approximate area of the plot by using trapezoidal rule.

(b) Solve the differential equation $(x+1)\dfrac{dy}{dx} - ny = e^x(x+1)^{n+1}$.

(10 + 10 = 20 Marks)

Q.5 **(a)** Assume that the half life of the radium in a piece of lead is 1500 years. How much radium will remain in the lead after 2500 years?

(b) A particle of mass m is moving under the action of the forces $F_1 = -m\omega^2 x$, $F_2 = mF_0 t$, $F_3 = -2m\mu\dfrac{dx}{dt}$. Assuming that damping is small, set up and solve the equation of motion.

(10 + 10 = 20 Marks)

SECTION-B

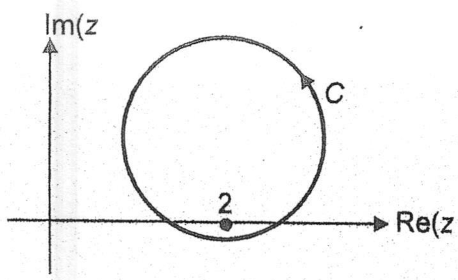
Q.6 **(a)** Find radius and interval of convergence of the series of $\sum_1^{\infty} \dfrac{n!\, x^n}{(2n)!}$.

(b) Prove that $(\sin x + i\cos x)^n = \cos n\left(\dfrac{\pi}{2} - x\right) + i\sin n\left(\dfrac{\pi}{2} - x\right), n \in Z$.

(10 + 10 = 20 Marks)

Q.7 **(a)** Construct the analytic function whose real part is $e^{-x}[(x^2 - y^2)\cos y + 2xy\sin y]$.

(b) Compute $\int_C \dfrac{z^2+2}{z(z^2-4)(z+4)}dz$, where C is the curve shown in the figure below:



(10+10=20 Marks)

Q.8 **(a)** Find the tangent and normal to the curve $x^2 - xy + y^2 = 7$ at the point $(-1,2)$.

(b) Find the curvature and torsion of the circular helix $\vec{r} = (a\cos u,\ a\sin u,\ bu)$.

(10 + 10 = 20 Marks)

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FOR RECRUITMENT TO THE POSTS OF
PROVINCIAL MANAGEMENT SERVICE, ETC -2021
CASE NO. 3C2022

SUBJECT: MATHEMATICS (PAPER-II)

TIME ALLOWED: THREE HOURS **MAXIMUM MARKS: 100**

NOTE:
- i. All the parts (if any) of each Question must be attempted at one place instead of at different places.
- ii. Write Q. No. in the Answer Book in accordance with Q. No. in the Q. Paper.
- iii. No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
- iv. Extra attempt of any question or any part of the question will not be considered.

NOTE: **Attempt any five questions. All questions carry equal marks. Calculator is not allowed (Programmable)**

Q.No.1

a) Let P_2 be a set of all polynomials having degree at most equal to 2 and $T: P_2 \rightarrow P_2$ a linear transformation such that $T(3-5x) = 2$ and $T(1-x+2x^2) = 1+x$.
Find $T(6-11x - 3x^2)$.

b) Let V and W be two vector spaces over the same field F and T: V →W a linear transformation. Show that T is one-to-one if and only if Ker(T)= {0}.

(10 + 10 = 20 Marks)

Q.No.2

a) Find the eigen values and the corresponding eigen vectors of $A = \begin{bmatrix} -1 & 0 & 1 \\ 3 & 0 & -3 \\ 1 & 0 & -1 \end{bmatrix}$.

b) If possible, find a matrix P such that $P^{-1}\begin{pmatrix} 1 & 3 \\ 2 & 2 \end{pmatrix} P$ is a diagonal matrix.

(10 + 10 = 20 Marks)

Q.No.3

a) Using Gram-Schmidt process, orthonormalize the set $\{1, t, t^2\}$ on the interval $[-1, 1]$ with the following inner product $< x.y >= \int_{-1}^{1} x(t).y(t)\, dt$.

b) Show that $C[a, b] = \{ f: [a, b] \rightarrow R \text{ where } f \text{ is a continuous function}\}$ is not an inner product space.

(10 + 10 = 20 Marks)

Q.No.4

a) Let (X_1, d_1) and (X_2, d_2) be two metric spaces. If (X_1, d_1) is a complete metric space and isometric with (X_2, d_2), show the (X_2, d_2) is a complete metric space.

b) Let (X, d) be a metric space and T a collection of all subsets of X. Show that T is a topology on X.
Recall that, a set U in X is open if for each $\epsilon\, U$, there exists a real number $r > 0$ such that $x \in S_r(x) \subseteq U$.

(10 + 10 = 20 Marks)

Q.No.5

a) If H is a non-empty finite subset of a group G and H is closed under multiplication, then show that H is a subgroup of G.

b) If G is a finite group and g ϵ G, then show that order of g divides the order of G.

(10 + 10 = 20 Marks)

Q.No.6

a) If $\emptyset$ is a homomorphism of group G_1 into a group G_2 with kernel K, then by proving $\emptyset\,(x^{-1}) = [\emptyset(x)]^{-1}$, show that K is a normal subgroup of G.

b) If G is a group, then show that the set of all automorphisms of G is a group.

(10 + 10 = 20 Marks)

Q.No.7

a) Show that a finite integral domain is a Field.

b) If $v_1, v_2, \dots v_n$ are vectors in a vector space V, then show that either they are linearly independent or some Vector v_m is a linear combination of $v_1, v_2, \dots v_{m-1}$.

(10 + 10 = 20 Marks)

Q.No.8

a) Find a basis and dimension of the span of $\{[1,4,-1,3],[2,1,-3,-1],[0,2,1,-5]\}$

b) Give at least four examples of infinite dimensional vector spaces and show that the set of all polynomials in x with real coefficients is an infinite dimensional vector space.

(10 + 10 = 20 Marks)